Dynamically Optimal R&D Subsidization

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Abstract: This paper characterizes the optimal time path of R&D and capital subsidization. Starting from the steady state under current R&D subsidization in the US, the R&D subsidy should significantly jump upwards and then slightly decrease over time. There is a small loss in welfare, however, from immediately setting the R&D subsidy to its optimal long run level, compared to a time-varying R&D subsidy. The results do not depend on the financing scheme, namely lump sum taxation or factor income taxation. The optimal capital subsidy is time-varying under factor income taxation, but time-invariant when subsidies are financed by lump sum taxes.

Key words: R&D subsidy; Transitional dynamics; Semi-endogenous growth; Welfare.

JEL classification: H20, O30, O40.

Introduction

A large body of empirical evidence suggests that the social return to R&D exceeds the private return by a wide margin (e.g. Scherer, 1982; Grilichis and Lichtenberg, 1984).[1] By focussing on the long run, a similar finding is typically derived from calibrated endogenous growth models. In fact, positive externalities from R&D seem to substantially outweigh negative externalities (e.g. Romer, 2000; Jones and Williams,

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[1]See also Jones and Williams (1998) for a discussion of the empirical literature in the light of endogenous growth theory.

2000; Steger, 2005; Grossmann, Steger and Trimborn, 2010). For instance, Jones and Williams (2000) argue on basis of a semi-endogenous growth model in the spirit of Jones (1995), that the optimal long run R&D effort may be about twice the decentralized R&D effort.

One widely discussed policy implication from the apparent R&D underinvestment problem is to provide R&D subsidies to innovating industries. According to empirical evidence, such tax incentives indeed seem to be successful in stimulating R&D investments (e.g. Bloom, Griffith and van Reenen, 2002). The question is thus not so much whether or not R&D subsidies should be provided to innovating industries. One rather needs to know to what extent R&D should be subsidized and how R&D subsidization should change over time as the economy develops.

The existing literature has examined optimal R&D subsidization by either focussing on static models or exclusively on the steady state in dynamic models.[2] However, as it is well known, in R&D-based models of economic growth the speed of convergence is typically low.[3] Thus, any attempt to provide a careful policy recommendation requires to investigate the entire time path of the first-best R&D subsidy along the transition to the steady state. This is true even for advanced economies, which may have come close to their decentralized steady state. The reason is that current R&D subsidy rates may be far away from their long run social optimum. In this case, implementing the optimal long run policy would induce transitional dynamics with potentially long lasting adjustment to the new steady state. Hence, it is a priori not clear whether looking at the long run optimal R&D subsidization is indeed meaningful when it comes to providing careful policy recommendations.

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[2] One exception is Arnold (2000), who derives the optimal production and R&D subsidy for Romer's (1990) model. Nuño (2012) analyzes optimal R&D subsidies during business cycles rather than in an endogenous growth context.

[3] See, for instance, Steger (2003) who examines the speed of convergence in the endogenous growth model of Segerstrom (1998). Grossmann, Steger and Trimborn (2010) find a similar pattern for an extended Jones (1995) model.

This paper attempts to characterize the dynamically optimal, i.e. first-best, investment subsidies on R&D and capital costs on the basis of a calibrated, semi-endogenous R&D-based growth model with accumulation of both knowledge and physical capital. We deliberately choose to analyze the optimal dynamic R&D tax in a standard framework, i.e. a version of the semi-endogenous growth model by Jones (1995). The reason is that, for a first numerical analysis of a dynamically optimal R&D policy, we consider it an advantage to be able to analytically derive the steady state (decentralized equilibrium and social planner solution) and to draw on a deep understanding of its properties (e.g. Jones, 2005). Thus, given the stylized nature of the model, the main contribution of the paper is not so much to find the magnitude of optimal R&D subsidies, which is inherently difficult and potentially model-specific.[4] Rather, our goal is to study a well-known, tractable framework which allows us to focus on the following questions. First, how does the optimal time path of the R&D subsidy and important allocation variables depend on the initial conditions of the economy? Second, and maybe most original, how large is the welfare loss, in terms of permanent consumption-equivalent changes, when we implement the long run optimal (time-invariant) policy rather than the dynamically optimal one (i.e. the time-varying, first-best policy). This question may be of high relevance for real-life policy. Policy makers may be constrained to set policy instruments at time-invariant levels for the sake of simplicity. We would like to have an idea about the magnitude of the loss from such a political constraint. Moreover, policy makers do not know at which point along the transition path the economy is located.

Our results indicate that the optimal R&D subsidy is time-dependent and adjusts monotonically towards the steady state. Whether it should decrease or increase over time depends on the gap in the knowledge stock and the capital stock to their optimal

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[4]For instance, we neglect factor reallocation costs, possibly leading to an overstatement of optimal R&D subsidies.

steady state levels. For instance, if the knowledge stock is far away from the optimal steady state relative to the capital stock, the optimal R&D subsidy should initially start above its optimal long run level and decrease over time when the economy approaches the steady state. For the US, our analysis suggests that the R&D subsidy should jump up significantly from its current level and then slightly decrease over time.

The most important and striking result from our analysis is that the welfare loss from immediately setting the R&D subsidy to its optimal long run level (i.e. the constrained optimum) is rather small compared to the case where the dynamically optimal policy path is implemented. In other words, the error of neglecting the transitional dynamics when designing the optimal R&D subsidy is small, despite the fact that the speed of convergence to the steady state is fairly low.[5]

We start out with the standard assumption that investment subsidies are financed by a lump-sum tax. Alternatively, we assume that they are financed by a mix of factor income taxes. As is well known, capital income taxation distorts capital accumulation from the supply-side. We show that, as a consequence, the optimal subsidy on capital costs of firms depends on the tax rate on bond yields. Moreover, we find that it should change over time, whereas with lump sum taxes the capital subsidy should be time-invariant. When implementing the optimal long run subsidy rates for R&D *and* capital costs under factor income taxation rather than the dynamically optimal policy program, the loss of intertemporal welfare is somewhat higher than in the case of lump sum taxation, but still small.

Dynamic optimal tax problems have been extensively studied in neoclassical growth models with fixed government expenditure (e.g. Judd, 1985; Chamley, 1986). For instance, it is well known that the optimal Ramsey-tax on capital income is highly time-variant in a closed-economy neoclassical growth framework with endogenous labor

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[5]Our analysis suggests that after a policy reform in the US which implements the optimal R&D subsidization it takes more than 200 years to close half of the gap of per capita consumption to the new steady state.

supply. Early in the transition, capital should be taxed at high rates due to the lump-sum tax character of taxing the existing capital stock. However, in an infinite horizon context the optimal capital tax becomes zero when the economy approaches the steady state (i.e. required tax revenue is exclusively financed by labor income taxation).[6] In an endogenous growth model there is no analogy with respect to R&D subsidies.[7] Rather than studying a second-best Ramsey-tax problem, we seek for dynamically optimal Pigouvian subsidies which eliminate inefficient investment incentives into R&D and physical capital under laisser-faire. These are distorted, for instance, by intertemporal knowledge spillovers. We show that the first-best allocation can be restored with two linear (though potentially time-varying) policy instruments in our framework, i.e. subsidies on R&D and capital costs of firms. Somewhat surprising, this result still holds when these subsidies are financed by a distortionary, linear tax on capital income. In other words, the efficiency of capital accumulation can be fully restored with one (possibly time-varying) instrument despite multiple sources of market failures which distort capital accumulation, i.e. the market power of capital good producers and capital income taxation.

Our paper is closely related to Arnold (2000), who shows for Romer's (1990) model how the first best allocation can be achieved by subsidies on intermediate good producers and R&D. His qualitative findings parallel ours. A constant subsidy for intermediate good producers combined with a time-varying optimal R&D subsidy restores the first best allocation. While Arnold explores optimal subsidies qualitatively, he abstains from numerical analysis and does not investigate to which degree implementing a constant R&D subsidy rather than a dynamically optimal one affects welfare. Also related to our paper are studies which derive the long run optimal R&D subsidy in an endogenous

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[6]The result is potentially modified in overlapping-generation models (e.g., Conesa, Kitao and Krueger, 2009).

[7]Moreover, the transitional dynamics are more complex when, in addition to physical capital accumulation, R&D-based innovations are a second engine of growth.

growth framework. For instance, Sener (2008) studies an endogenous growth model without scale effects where the steady state growth rate depends on the R&D subsidy. Calibration exercises indicate that the optimal steady state R&D subsidy should range between 5 and 25 percent. More recently, Nuño (2011) analyzes optimal long run investment subsidies in a Schumpeterian growth model with business cycles. However, these contributions abstract from transitional dynamics when deriving the optimal policy scheme. Grossmann, Steger and Trimborn (2010) extend the semi-endogenous growth framework of Jones and Williams (2000) to capture distortions from the tax-transfer system. Their results suggest that in the long run firms should be allowed to deduct at least twice the R&D costs from sales revenue to calculate corporate income. This policy recommendation still holds when the intertemporal welfare gain from a policy reform is maximized, provided that the subsidy rates are constrained to be time-invariant. In this paper, we relax the latter restriction and analyze the socially optimal transitional dynamics.

The paper is organized as follows. Section 2 presents an endogenous growth model with linear subsidies on capital costs and R&D costs, derives the dynamic system in decentralized equilibrium and analytically characterizes the dynamically optimal capital and R&D subsidization. Section 3 presents and discusses the calibration of the model. Based on the calibration strategy, section 4 numerically analyzes the socially optimal evolution of important allocation variables and the R&D subsidy under alternative initial conditions. Most importantly, it also compares the optimal dynamics with the one resulting from implementing the time-invariant, optimal long run R&D subsidy from the start. Section 5 allows for distortionary taxation. The last section concludes.

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Basic Model

Consider the following continuous-time model with semi-endogenous economic growth, based on Jones (1995). There is a homogenous final output good with price normalized to unity. Final output is produced under perfect competition according to

$$Y = (L^Y)^{1-\alpha} \int_0^A (x_i)^\alpha di, \tag{1}$$

$0 < \alpha < 1$, where L^Y is labor input in the manufacturing sector, A is the mass ("number") of intermediate goods and x_i denotes the quantity of intermediate good i. (Time index t is omitted whenever this does not lead to confusion.) The number of varieties, A, expands through horizontal innovations, protected with patent rights of infinite length. As usual, A is interpreted as the economy's stock of knowledge. $A_0 > 0$ is given. The labor market is perfect.

In each sector i there is one firm $-$ the innovator or the buyer of a blueprint for an intermediate good $-$ which can produce good x_i with a one-to-one technology: one unit of foregone consumption (capital) can be transformed into one unit of output. Capital depreciates at rate $\delta \geq 0$. Capital supply in the initial period, K_0, is given. The capital market is perfect.

Moreover, in each sector i there is a competitive fringe which can produce a perfect substitute for good i (without violating patent rights) but is less productive in manufacturing the good: one unit of output requires κ units of capital; $1 < \kappa \leq 1/\alpha$.[8]

There is free entry into the R&D sector. Ideas for new intermediate goods are

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[8]See Aghion and Howitt (2005), among others, for similar way of capturing a competitive fringe. Allowing for a competitive fringe is useful to calibrate the mark-up factor over marginal costs and to disentangle price setting power from output elasticities.

generated according to

$$\dot{A} = \tilde{\nu} A^{\phi} L^{A}, \text{ with } \tilde{\nu} \equiv \nu \left(L^{A} \right)^{-\theta}, \tag{2}$$

where L^{A} is the labor input in the R&D sector, $\nu > 0$, $\phi < 1$, $0 \leq \theta < 1$. $\tilde{\nu}$ is taken as given in the decision of the representative R&D firm. That is, R&D firms perceive a constant returns to scale R&D technology, although the social return to higher R&D input is decreasing when $\theta > 0$. The wedge between the private and social return may arise because firms do not take into account that rivals may work on the same idea such that there is redundant R&D in market equilibrium. θ measures the strength of such "duplication externality". If $\phi \neq 0$, there is a second R&D externality; $\phi > 0$ captures a standard "standing on shoulders" effect, whereas the case $\phi < 0$ reflects by contrast that R&D productivity declines with the number of preceding innovations (possibly because the most obvious innovations are detected first; see Jones, 1995, for a discussion).[9]

There is an infinitely-living, representative dynasty with initial per capita wealth, $a_0 > 0$. Household size, N, grows with constant exponential rate, $n \geq 0$. N_0 is given and normalized to unity. Preferences are represented by the standard utility function

$$U = \int_{0}^{\infty} \frac{(c_t)^{1-\sigma} - 1}{1 - \sigma} e^{-(\rho - n)t} dt, \tag{3}$$

$\sigma > 0$, where c is consumption per capita. Households take factor prices as given.

The government may subsidize both R&D costs (R&D sector) and capital costs (intermediate goods sector). In the basic model, we assume that both subsidies are financed by lump-sum taxes (T) on households. In section 5, we allow for distortionary

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[9]In his seminal contribution on endogenous technical change, Romer (1990) assumes $\phi = 1$. This assumption has been modified by Jones (1995) as $\phi = 1$ implies that the economy's growth rate depends on the aggregate human capital level ("strong scale effect"), a prediction which seems to be largely inconsistent with the data. We therefore follow Jones (1995).

taxation. The subsidy rates are independent of total costs at one point in time (i.e. linear), but possibly time-variant. They are denoted by s_A (R&D) and s_K (capital), respectively.

Let w and r denote the wage rate and the interest rate, respectively. Financial wealth per individual, a, accumulates according to

$$\dot{a} = (r - n)a + w - c - T, \tag{4}$$

with a_0 being given.

It turns out that, for the transversality conditions to hold, we have to restrict the parameter space such that

$$\rho - n + (\sigma - 1)g > 0 \text{ with } g \equiv \frac{(1 - \theta)n}{1 - \phi}. \tag{A1}$$

As will become apparent, g is the economy's long run growth rate in decentralized equilibrium as well as in social planning optimum. According to (2), $\dot{A}/A = \nu \left(L^A\right)^{1-\theta} A^{\phi-1}$. Thus, if R&D labor L^A grows with population growth rate n (which we confirm) we immediately see that $\dot{A}/A = g$ in steady state. Moreover, substitute $c_t = c_0 e^{gt}$ into (3) to confirm that utility U is finite if and only if $\rho - n + (\sigma - 1)g > 0$ holds. We maintain assumption A1 throughout.

Market Equilibrium

We first derive the decentralized equilibrium and show how the steady state allocation of labor and the steady state savings rate (equal to the investment rate) depends on policy parameters.

We start with intermediate goods producers. Note that $r + \delta$ is the user cost per unit of capital for an intermediate good firm. As one unit of capital is required for one

unit of output, if the government subsidizes capital costs at rate s_K, producer i has profits

$$\pi_i = [p_i - (1 - s_K)(r + \delta)]\, x_i, \tag{5}$$

where p_i is the price of good i. According to (1), the inverse demand function for intermediate good i reads $p_i = \alpha (L^Y / x_i)^{1-\alpha}$.

Profit maximization implies that the optimal price of each firm i is given by

$$p_i = p = \kappa(1 - s_K)(r + \delta). \tag{6}$$

To see this, note that a firm which owns a blueprint would choose a mark-up factor which is equal to $1/\alpha \geq \kappa$ if it were not facing a competitive fringe. Moreover, the competitive fringe would make losses at a price lower than $\kappa(1 - s_K)(r + \delta)$. Thus, each firm i sets the maximal price allowing it to remain monopolist. We can substitute (6) into the inverse demand function and solve for x_i to obtain output

$$x_i = x = \left(\frac{\alpha}{\kappa(1 - s_K)(r + \delta)} \right)^{\frac{1}{1-\alpha}} L^Y. \tag{7}$$

Substituting (7) into (1) gives

$$Y = A \left(\frac{\alpha}{\kappa(1 - s_K)(r + \delta)} \right)^{\frac{\alpha}{1-\alpha}} L^Y \tag{8}$$

Moreover, as the total amount of physical capital is $K \equiv \int_0^A x_i di = Ax$, the capital-output ratio is given by

$$\frac{K}{Y} = \frac{\alpha}{\kappa(1 - s_K)(r + \delta)}. \tag{9}$$

Thus, if the interest rate r is stationary in the long run, the total capital stock and aggregate income grow at the same rate along a balanced growth path.

Let us denote the present discounted value of the profit stream generated by an

innovation by P^A (being equal to the price an intermediate good producer pays to the R&D sector for a new blueprint and to the stock market evaluation of a firm). In equilibrium, there are no arbitrage possibilities in the capital market. Noting that all intermediate goods producers have the same profit due to the symmetry in their sector, i.e. $\pi_i = \pi$ for all i, this implies the standard capital market equilibrium condition

$$\frac{\dot{P}^A}{P^A} + \frac{\pi}{P^A} = r. \tag{10}$$

Let us define $\tau \equiv 1 - s_A$. In the R&D sector, under R&D subsidy rate s_A, a representative firm maximizes

$$\Pi = P^A \underbrace{\tilde{\nu} A^\phi L^A}_{= \dot{A}} - \tau w L^A, \tag{11}$$

taking A, $\tilde{\nu}$ and prices as given. That is, in equilibrium, $\Pi = 0$.

The household's problem is to solve

$$\max_{\{c_t\}} \int_0^\infty \frac{(c_t)^{1-\sigma} - 1}{1 - \sigma} e^{-(\rho-n)t} dt \text{ s.t. (4)}, \lim_{t \to \infty} a_t e^{-\int_0^t (r_s - n) ds} \geq 0. \tag{12}$$

The household chooses the optimal consumption path, where savings are supplied to the financial market.

Definition. A market equilibrium in this economy consists of time paths for the quantities $\{L_t^A, L_t^Y, c_t, \{x_{it}\}_{i=0}^A, a_t, Y_t, K_t, A_t\}_{t=0}^\infty$ and prices $\{P_t^A, \{p_{it}\}_{i=0}^A, w_t, r_t\}_{t=0}^\infty$ such that

1. final goods producers, intermediate goods producers and R&D firms maximize profits,

2. households maximize intertemporal welfare,

3. the capital resource constraint $\int_0^A x_{it}di = K_t$ holds,

4. the capital market equilibrium condition, equ. (10), holds,

5. the labor market, the intermediate goods market, and the financial market clear,[10]

6. the government budget is balanced.

We define per capita measures $l^A \equiv L^A/N$, $l^Y \equiv L^Y/N$, $k \equiv K/N$, $y \equiv Y/N$ and $p^A \equiv P^A/N$. Clearing of the financial market requires

$$aN = K + P^A A. \tag{13}$$

Moreover, in labor market equilibrium,

$$l^A + l^Y = 1. \tag{14}$$

Along a balanced growth path, all variables grow at a constant (possibly zero) rate. We descale those variables which turn out to grow, as in Jones (1995), with rate $g = \frac{(1-\theta)n}{1-\phi}$ in steady state and define "adjusted" levels $\tilde{A} = A/N^{\frac{1-\theta}{1-\phi}}$, $\tilde{k} = k/N^{\frac{1-\theta}{1-\phi}}$ and $\tilde{c} = c/N^{\frac{1-\theta}{1-\phi}}$. Proposition 1 (below) presents the full dynamical system which governs the evolution of the market equilibrium and its steady state. Steady state levels are indicated by superscript (*).

Proposition 1. (Dynamic system for market equilibrium)

(i) Under lump-sum taxation, given the time paths of $\tau = 1 - s^A$ and s_K, the evolution of $\tilde{A}$, p^A, $\tilde{k}$, $\tilde{c}$, r, $l^A = 1 - l^Y$ is governed by the following dynamic system

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[10] According to Walras' law, the final goods market then clears as well.

(together with appropriate boundary conditions)

$$\frac{\dot{\tilde{A}}}{\tilde{A}} = \nu \tilde{A}^{\phi-1} \left(l^A\right)^{1-\theta} - g, \tag{15}$$

$$\frac{\dot{p}^A}{p^A} = r - n - \frac{(\kappa-1)\left(\alpha/\kappa\right)^{\frac{1}{1-\alpha}}\left(1-l^A\right)}{[(1-s_K)(r+\delta)]^{\frac{\alpha}{1-\alpha}}p^A}, \tag{16}$$

$$\frac{\dot{\tilde{k}}}{\tilde{k}} = \left(\frac{\tilde{A}(1-l^A)}{\tilde{k}}\right)^{1-\alpha} - \frac{\tilde{c}}{\tilde{k}} - \delta - n - g, \tag{17}$$

$$\frac{\dot{\tilde{c}}}{\tilde{c}} = \frac{r-\rho}{\sigma} - g, \tag{18}$$

$$r + \delta = \frac{\alpha}{\kappa(1-s_K)}\left(\frac{\tilde{A}(1-l^A)}{\tilde{k}}\right)^{1-\alpha}, \tag{19}$$

$$p^A \nu \tilde{A}^{\phi-1}(l^A)^{-\theta} = \tau(1-\alpha)\left(\frac{\tilde{k}}{\tilde{A}(1-l^A)}\right)^{\alpha}. \tag{20}$$

(ii) In the long run, there exists a unique balanced growth equilibrium, where

$$r = \sigma g + \rho \equiv r^*, \tag{21}$$

$$l^A = \frac{1}{\frac{\tau(1/\alpha-1)(\sigma g+\rho-n)}{(1-1/\kappa)g} + 1} \equiv (l^A)^*, \tag{22}$$

$$\tilde{A} = \left(\frac{\nu\left((l^A)^*\right)^{1-\theta}}{g}\right)^{\frac{1}{1-\phi}} \equiv \tilde{A}^*, \tag{23}$$

$$p^A = \frac{(\kappa-1)\left(\alpha/\kappa\right)^{\frac{1}{1-\alpha}}\left(1-(l^A)^*\right)}{[(r^*+\delta)(1-s_K)]^{\frac{\alpha}{1-\alpha}}(r^*-n)} \equiv (p^A)^*, \tag{24}$$

$$\tilde{k} = \tilde{A}^*(1-(l^A)^*)\left(\frac{\alpha}{\kappa(1-s_K)(r^*+\delta)}\right)^{\frac{1}{1-\alpha}} \equiv \tilde{k}^*, \tag{25}$$

$$\tilde{c} = (\tilde{k}^*)^{\alpha}(\tilde{A}^*)^{1-\alpha}(1-(l^A)^*)^{1-\alpha} - (\delta+n+g)\tilde{k}^* \equiv \tilde{c}^*. \tag{26}$$

In the long run, k, y, c and A grow at rate $g = \frac{(1-\theta)n}{1-\phi}$. The savings and investment

rate, $\mathfrak{s} \equiv 1 - \frac{c}{y}$, *is given by*

$$\mathfrak{s} = \frac{\alpha(n + g + \delta)}{\kappa(1 - s_K)(\sigma g + \rho + \delta)} \equiv \mathfrak{s}^*. \qquad (27)$$

Proof. See Appendix. ■

Like in Jones (1995), the growth rate of per capita income along a balanced growth path is independent of economic policy (in contrast to the level of income). Proposition 1 also implies that life-time utility (3) is finite if and only if assumption (A1) holds.

Moreover, Proposition 1 suggests that subsidizing physical capital does not affect the allocation of labor in long run equilibrium, but an increase in s_K raises the long run savings and investment rate, $\mathfrak{s}^*$. Similarly, an increase in the R&D subsidy rate s_A (i.e. a decline in τ) stimulates R&D activity of firms (i.e. $(l^A)^*$ increases); it does not, however, affect the long run equilibrium rate of investment in physical capital, $\mathfrak{s}^*$.

Social Planning Optimum

A social planner chooses a symmetric capital allocation across intermediate firms, i.e. $x_i = K/A$ for all i. Using this in production function (1) yields per capita output $(y = Y/N)$:

$$y = k^\alpha (A l^Y)^{1-\alpha}. \qquad (28)$$

Thus, using the goods market clearing condition $\dot{K} = Y - Nc - \delta K$, the capital stock per capita $(k = K/N)$ evolves according to

$$\dot{k} = k^\alpha (A l^Y)^{1-\alpha} - (\delta + n)k - c. \qquad (29)$$

Also note that the social planner takes R&D externalities into account such that the relevant knowledge accumulation technology is

$$\dot{A} = \nu A^{\phi}(Nl^A)^{1-\theta}. \tag{30}$$

The social planner's problem thus is to solve

$$\text{max (3) s.t. (14), (29), (30),} \tag{31}$$

and non-negativity constraints, where c, l^A, l^Y are control variables and k, A are state variables.

Comparing the social planning optimum to the decentralized equilibrium, we may ask if the two policy instruments, a subsidy to R&D and capital costs, can restore the first best optimum, in view of the following market failures. First, due to monopolistic competition as compared to marginal cost pricing, intermediate goods supply and therefore the demand for capital is inefficiently low. Thus, savings (and thus capital investment) may be too low, calling for a capital cost subsidy. Moreover, there are three sources of inefficient R&D incentives. The duplication externality ($\theta > 0$) promotes overinvestment in R&D, whereas a standing on shoulders effect ($\phi > 0$) promotes underinvestment. (In the case where $\phi < 0$, there is a force towards overinvestment.) Finally, innovators can only appropriate part of the economic surplus from raising the knowledge stock of the economy. To see this, first note that $x_i = x = \frac{K}{A} = \frac{K}{Y}\frac{Y}{A}$. Substituting this into (5) and using (6) and (9) implies that instantaneous profit of an intermediate goods firm reads $\pi = \alpha(1 - \frac{1}{\kappa})\frac{Y}{A}$. Moreover, according to (8), we have $\frac{\partial Y}{\partial A} = \frac{Y}{A}$. Since $\alpha(1 - \frac{1}{\kappa}) < 1$, the per-period profit π for an innovator is lower than the contribution of an additional blueprint to output, $\frac{\partial Y}{\partial A}$. In other words, there is a "surplus appropriability problem" which promotes underinvestment.

The next proposition shows that appropriately setting $\tau = 1 - s_A$ and s_K in the market economy can indeed implement the first best optimum. Moreover, it turns out that the optimal R&D subsidy is time-variant rather than being equal to the time-invariant long run optimum, whereas the optimal capital subsidy does not change over time. We will closely examine the implications of this insight below.

Proposition 2. (Social optimum) *Under lump sum taxation, the first-best optimal evolution of the economy can be supported by setting a time-invariant capital cost subsidy*

$$s_K = 1 - \frac{1}{\kappa} \equiv (s_K^{opt})^* \tag{32}$$

together with a time-variant R&D subsidy $s_A = 1 - \tau$, where the optimal τ, denoted by τ^{opt}, evolves according to

$$\frac{\dot{\tau}^{opt}}{\tau^{opt}} = \left[\left(1 - \theta - \frac{1}{\tau^{opt}} \frac{1 - 1/\kappa}{1/\alpha - 1} \right) \left(\frac{1}{l^A} - 1 \right) + \phi \right] \nu \tilde{A}^{\phi - 1} \left(l^A \right)^{1 - \theta} \tag{33}$$

with terminal condition (long-run optimal R&D subsidy)

$$\tau = \frac{1 - 1/\kappa}{1/\alpha - 1} \frac{(\sigma - 1)g + \rho - \theta n}{(1 - \theta)(\sigma g + \rho - n)} \equiv (\tau^{opt})^*. \tag{34}$$

Proof. See Appendix. ∎

A higher mark up factor κ drives, in absence of policy intervention, a bigger wedge between the equilibrium investment rate and the socially optimal investment rate, calling for a higher subsidy on capital costs to ensure the first best savings and investment rate. The optimal subsidy rate which implements the first-best is time-invariant under lump sum taxation.[11] At the same time, if κ rises, the surplus appropriability problem, which promotes sub-optimally low investment in R&D, becomes less severe, such that

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[11] Notice that market power of intermediate good producers precludes the case $s_K > (s_K^{opt})^*$.

the optimal long run R&D subsidy, $(s_A^{opt})^* := 1 - (\tau^{opt})^*$, decreases. This suggests that stronger patent protection should be accompanied by lower R&D subsidies. As long run growth is policy-independent and the dynamically optimal Pigouvian subsidies implement the first best allocation, Proposition 2 implies that the long-run growth rate in social optimum coincides with the one arising in decentralized equilibrium.

The important question we examine below is whether the optimal R&D subsidy should increase or decrease over time, given that the capital cost subsidy is set optimally. Moreover, as previous studies of optimal R&D subsidies have exclusively focussed on the long run, it is interesting to compare the evolution of important variables (including welfare) under the first-best (i.e. time-varying) R&D subsidy and the optimal steady state (i.e. the time-invariant) R&D subsidy. As both requires numerical analysis, we need to calibrate the model first.

Calibration

We calibrate the model for the US economy. The strategy is to match steady state values of important variables. First, g is set to the average US GDP per capita growth rate for the period 1990-2004. Taking data from the Penn World Tables (PWT) 6.2 (Heston, Summers and Baten, 2006), we find that approximately $g = 0.02$.[12] Moreover, for the same period and again from PWT 6.2, the average population growth rate is approximately $n = 0.01$.

We use measures for the investment rate and the capital-output ratio to calibrate the depreciation rate of physical capital as follows. The investment share is given by $\mathfrak{s} = (\dot{K} + \delta_K K)/Y = (\dot{K}/K + \delta)K/Y$. Using $\dot{K}/K = n + g$ and solving for δ yields

$$\delta = \frac{\mathfrak{s}}{K/Y} - n - g. \tag{35}$$

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[12] Averaging over some years takes out business cycle phenomena.

Averaging over the period 1990-2004, $\mathfrak{s}$ is equal to about 21 percent, according to PWT 6.2. For the capital-output ratio, K/Y, we take averages over the period 2002-2007 calculated from data of the US Bureau of Economic Analysis. The capital stock is measured by total fixed assets (private and public structures, equipment and software). When measuring K and Y in current prices, this gives us $K/Y = 3$. From (35), the evidence thus suggests $\delta = 0.04$, a standard value in the literature.

Moreover, the steady state interest rate is set to $r^* = 0.07$, which coincides with the real long-run stock market return estimated by Mehra and Prescott (1985).[13] According to (21), for $g = 0.02$, $r^* = 0.07$, and a typical value for the time preference rate of $\rho = 0.02$, one gets $\sigma = 2.5$.

We also assume that the capital cost subsidy rate, s_K, is at the optimal level, for two reasons: First, this allows us to focus on the consequences of deviating from the optimal path of the R&D subsidy rate. Second, one may argue that s_K actually is at the optimal level $(s_K^{opt})^*$ at present in the US. In line with estimates for the average mark up factor in the economy (e.g. Norrbin, 1993), setting $\kappa = 4/3$ implies $(s_K^{opt})^* = 1 - 1/\kappa = 0.25$. Now, given a rate of depreciation allowances for capital investments, s_d, and a corporate income tax rate, τ_c, the behaviorally relevant capital cost subsidy is $s_K = \frac{\tau_c s_d}{1 - \tau_c}$ (e.g. Grossmann, Steger and Trimborn, 2010). According to Devereux, Griffith and Klemm (2002), in the US, we approximately have $s_d = 0.75$. For large corporations, the federal US statutory corporate income tax rate is 35 percent (and about 39 percent including sub-governments).[14] For small corporations, it is 15 (with sub-governments, about 20) percent. We may thus base our calibration on $\tau_c = 0.25$, which together with $s_d = 0.75$ indeed implies $s_K = 0.25$.

Next, using (9), we find that $\kappa(1 - s_K) = 1$, $r = 0.07$, $\delta = 0.04$ and $K/Y = 3$

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[13] Jones and Williams (2000) argue that this rate of return is more appropriate for calibration of growth models than the risk-free rate of government bonds. Setting a lower level for r does not affect the results when comparing the optimal dynamic R&D subsidy with the optimal long run policy.

[14] See the OECD tax database.

implies $\alpha = 0.33$, which is a typical value of the output elasticity of capital used in the growth literature.[15]

Parameter ν is irrelevant for the steady state allocation of labor in both market equilibrium and social optimum. Moreover, for a given growth rate of per capita income (g) and population size (n), we can use the relationship between the standing on shoulders parameter (ϕ) and the duplication externality parameter (θ) which is implied by $g = \frac{(1-\theta)n}{1-\phi}$. Hence, from $g = 0.02$ and $n = 0.01$ we get

$$\phi = 0.5(1 + \theta). \tag{36}$$

This leaves us with one degree of freedom. We focus our discussion on the case where $\theta = 0.5$ (medium degree of duplication externality), which implies that $\phi = 0.75$. According to (34), this implies an optimal long run R&D subsidy rate, $(s_A^{opt})^* \equiv 1 - (\tau^{opt})^*$, of 81.5 percent.[16] In a model with corporate income taxation, assuming a corporate tax rate $\tau_c = 0.25$, this would mean that innovating firms should be allowed to deduct $1 + \frac{(1-\tau_c)s_A}{\tau_c} = 3.4$ times their R&D costs from sales revenue to compute the corporate income tax base (Grossmann, Steger and Trimborn, 2010). Our main results and conclusions are unchanged when we use other values for θ, as long as θ is not too high (obviously, for $\theta \to 1$ no R&D should be conducted). For instance, for $\theta = 0.25$ we find that 3.6 times the R&D costs and for $\theta = 0.75$ about three times the R&D costs should be deductible. The current US R&D subsidy rate is given by

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[15]We consider it to be an advantage of our calibration strategy that we do not have to assume a value for α, but infer it from observables. Although α is one minus the labor share of income in neoclassical models, we cannot use this standard argument in our context where labor is not only used in final goods production. Moreover, as discussed by Krueger (1999), measurement of the labor share is difficult and inevitably depends on strong assumptions.

[16]This may seem high at the first glance. However, it is in line with previous calibration exercises. For instance, Grossmann, Steger and Trimborn (2010) show that a R&D subsidy of that kind of magnitude is required to solve the R&D underinvestment problem which is identified by Jones and Williams (2000). Notice that the focus of this paper is not on the size of the optimal long run R&D subsidy, $(s_A^{opt})^*$, but the comparison with the dynamically optimal R&D subsidy.

$s_A = 0.066$ (OECD, 2009),[17] which means that in the US only 1.2 times the R&D costs are deductible. With $s_A = 0.066$, we find that the share of labor devoted to R&D, $(l^A)^*$, is about 4.2 percent for our calibration, according to (22). This corresponds to a steady state R&D intensity, wl^A/Y, of 2.9 percent.

Quantitative Analysis

In this section, we make use of Proposition 1 and 2 to examine, based on the calibration described above, the optimal time path of the R&D subsidy rate (s_A) and main allocation variables and compare the results to the case where the optimal steady state R&D subsidy $(s_A^{opt})^* = 1 - (\tau^{opt})^*$ is implemented from the start. We also discuss policy implications from our analysis for the US economy.

Despite the simplicity of the model in terms of steady state properties, the numerical analysis is challenging. As Proposition 1 and 2 suggest, to examine the evolution of the variables of interest along the transition to the steady state requires the solution of highly dimensional, non-linear differential equation systems. We are able to do so by applying a novel numerical procedure which was suggested recently by Trimborn, Koch and Steger (2008). The essence of the method is described in appendix.

The Role of Initial Conditions

Panel (a) of Fig. 1 shows how the optimal level of $\tau = 1 - s_A$ evolves over time when the adjusted capital stock per capita $(\tilde{k})$ and/or the adjusted stock of knowledge $(\tilde{A})$ start below their optimal long run levels. Denote the optimal long run levels of $\tilde{A}$ and $\tilde{k}$

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[17]OECD (2009) reports a R&D subsidy rate $RDTS = 1 - Bindex$, where the so-called B-index is given by $Bindex = (1 - \Xi)/(1 - \tau_c)$, with τ_c being the statutory corporate income tax rate and Ξ the net present discounted value of depreciation allowances, tax credits and special allowances on R&D assets. We have $\Xi = \tau_c(1 + s_R)$, where s_R is the subsidy rate at which R&D costs which can be deducted from pre-tax profits. Thus, $RDTS = \tau_c s_R/(1 - \tau_c)$. Grossmann et al. (2010) show that $RDTS$ is equal to the behaviorally relevant R&D subsidy rate, s_A.

by $(\tilde{A}^{opt})^*$ and $(\tilde{k}^{opt})^*$, respectively, which result from setting optimal long run subsidy rates. For instance, suppose the initial knowledge stock is at 50 percent of its optimal long run level and the capital stock is initially optimal, which is *Scenario (i)* in Fig. 1. That is, $\tilde{A}_0 = 0.5(\tilde{A}^{opt})^*$ and $\tilde{k}_0 = (\tilde{k}^{opt})^*$. Then τ should start below the long run optimum, $(\tau^{opt})^*$, and increase over time. That is, the optimal R&D subsidy rate, $s_A^{opt} = 1 - \tau^{opt}$, should be high initially in view of the high initial gap in the knowledge stock ("knowledge gap") and decrease over time when $\tilde{A}$ comes closer to the optimal steady state level $(\tilde{A}^{opt})^*$.[18] Quantitatively, however, the variation in s_A^{opt} over time is small.

<FIGURE 1>

A qualitatively similar evolution is induced for the fraction of R&D labor, l^A, as displayed by the solid line in panel (b) for *Scenario (i)*. In this scenario the social planner reallocates labor in favor of R&D to close the initial knowledge gap, as seen in panel (b). This implies a drop in final output production and, holding the saving rate constant, would also imply a low level of consumption. To achieve a comparably smooth consumption path, however, the social planner reduces the savings rate, $\mathfrak{s}$, in parallel to high R&D investment. Subsequently, $\mathfrak{s}$ rises quite quickly over time (panel (d)). Panel (c) shows the evolution of (adjusted) per capita consumption level ($\tilde{c}$), discussed below.

In panels (b)-(d) of Fig. 1, we also compare the time paths of the allocation variables (l^A and $\mathfrak{s}$) and consumption under dynamically optimal R&D subsidization (solid lines) with the ones under the optimal steady state R&D subsidy (dashed lines). A time-invariant R&D subsidy rate which is set at its long run level may be referred to

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[18] A gap expresses the proportional difference between the initial value of the state variable under consideration and its socially optimal steady state value.

as "constrained optimal policy".[19] The constraint captures the potential difficulty to write tax laws which specify how policy rates change over time as well as the difficulty to know at which point along the transition path the economy is located. We see rather small differences in the respective time paths. It should also be observed that the difference between the first-best solution (solid line) and the solution under the constrained optimal steady state policy (dashed line) appears small. This raises the question about the welfare loss which results from implementing the constrained optimal steady state policy rather than dynamically optimal R&D subsidy path. The intertemporal welfare difference from the two associated consumption streams is expressed in terms of a (hypothetical) permanent loss of consumption ($\Delta\tilde{c}$) from implementing the constrained optimal policy program, in percent of the optimal long run consumption ($(\tilde{c}^{opt})^*$). (See appendix for details.) Formally, we denote $LOSS := \Delta\tilde{c}/(\tilde{c}^{opt})^*$. We find that the magnitude of $LOSS$ is small. For *Scenario (i)*, it is merely about 0.37 per mill. This result is in line with our finding that, although an unconstrained social planner would like to reduce the R&D subsidy rate as the knowledge gap narrows, quantitatively the initial dynamically optimal R&D subsidy rate is close to the optimal long run value, $(s_A^{opt})^*$. Intuitively, the result is an implication of the standard consumption-smoothing motive which prevents the social planner to close the knowledge gap too fast at the cost of lower consumption early on.

Fig. 1 also contains the scenarios where only the capital stock is at 50 percent of its optimal long-run level (*Scenario (ii)*) and the scenario where both the capital and the knowledge stock are at 50 percent of their long run levels (*Scenario (iii)*). In *Scenario (ii)*, the dynamically optimal R&D subsidy rate and the fraction of labor devoted to R&D both increase over time (i.e. τ^{opt} decreases and l^A increases). Moreover,

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[19]There is an alternative notion of a constrained optimum, where the welfare gain from a policy reform is maximized under the constraint that the R&D subsidy is constant over time (but taking into account the transition phase, assuming the economy is initially in steady state under the status quo policy). As shown in Grossmann, Steger and Trimborn (2010), a R&D policy which is optimal in this sense is very close to the optimal long run policy.

the savings rate ($\mathfrak{s}$) decreases over time. Hence, these variables qualitatively follow the opposite paths than in *Scenario (i)*, since now the focus of the social planer is on capital accumulation during the transition. In *Scenario (iii)*, neither the allocation variables (l^A and $\mathfrak{s}$) nor the first-best R&D subsidy change much over time and are close to their steady state values right from the start. This is unsurprising given the contrary evolution of these variables over time in *Scenario (i)* and *(ii)*. In *Scenario (iii)* the gaps in both stock variables (knowledge and capital) have to be closed simultaneously. Fig. 1 thus suggests that when the gap (i.e. the proportional difference between the initial value of the state variable and its socially optimal steady state) in knowledge is large relative to the gap in the capital stock, then in the beginning one should, relative to the steady state optimum, save little and invest much in R&D via a high R&D subsidy, whereas the opposite holds when the initial conditions constitute a relatively large gap in the capital stock.

Interestingly, the welfare loss from not choosing the dynamically optimal R&D subsidy but the constrained optimal steady state policy is again very small in *Scenario (ii)* and *(iii)*. The equivalent permanent loss in the per capita consumption level is 0.03 and 0.05 per mill, respectively, and thus even smaller than in *Scenario (i)*.

We now consider sensitivity analysis with respect to the intertemporal elasticity of substitution, $1/\sigma$, and the time preference rate, ρ, in two ways. First, both σ and ρ are varied together such that we still end up with a steady state interest rate $r^* = 0.07$. Secondly, we vary σ and ρ separately, which alters r^*. Moreover, we consider changes in the extent of duplication externality θ, and the strength of the "standing on shoulders" effect ϕ simultaneously to fulfill (36), i.e. we maintain the steady state growth rate $g = 0.02$. Finally, the mark up factor is alternatively set to $\kappa = 1.1$.[20] Tab. 1 presents these consumption equivalent losses in per mill ($LOSS$), for the three *Scenarios (i)-(iii)*

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[20]Recall that the optimal capital cost subsidy is always at its (time-invariant) optimal level. Thus, in the last column in Tab. 1, $\kappa = 1.1$ implies an optimal capital subsidy $(s_K^{opt})^* = 1 - 1/\kappa = 1/11$ instead of $(s_K^{opt})^* = 0.25$ in the other parameter sets.

under alternative parameter sets.

<TABLE 1>

Our sensitivity analysis suggests that the welfare loss ($LOSS$) from not implementing the dynamically optimal R&D subsidy is generally low, i.e. does not vary much with alternative parameter sets. To gain intuition, consider the time path of (adjusted) consumption, $\tilde{c}$, with initial conditions $\tilde{A}_0 = 0.5(\tilde{A}^{opt})^*$, $\tilde{k}_0 = (\tilde{k}^{opt})^*$ (*Scenario (i)*). In Fig. 2, we see the paths of $\tilde{c}$ for the baseline parameter set (i.e. $\sigma = 2.5$, $\rho = 0.02$) and the one where $\sigma = 1.5$, $\rho = 0.04$ under the two policy schemes (the solid lines indicate the paths of $\tilde{c}$ under the dynamically optimal policy, whereas the dashed line shows the path of $\tilde{c}$ under the constrained optimal policy for the baseline set of parameters and the dotted line gives the path of $\tilde{c}$ under the constrained optimal policy for $\sigma = 1.5$, $\rho = 0.04$). It becomes evident that for both parameter sets, the dynamically optimal R&D policy implies slightly lower consumption in the beginning than under the constrained optimal policy. As we know from Fig. 1, this results from a higher R&D subsidy than the long run optimal one in the transition phase, and therefore more labor allocated to R&D, to close the knowledge gap. Eventually, the consumption level in the first-best optimum overtakes the one under the constrained optimal R&D subsidy at a later stage of development. In *Scenario (i)*, $LOSS$ is thus related to the welfare loss from giving up consumption in the early transition phase under the first best policy compared to the constrained optimal policy in order to gain consumption later on.[21] Balancing gains and losses from the two alternative consumption paths results in a very small overall welfare loss. For both parameter sets in Fig. 2, the consumption paths cross approximately at the time when consumption starts to grow faster than

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[21] According to Fig.1 (panel (c)), in *Scenario (ii)*, where the capital stock is initially lower than in the optimal steady state, the social planer allows a higher consumption in an early transition phase in the dynamically optimal policy at cost of lower consumption later on.

at the steady state growth rate g (i.e. $\tilde{c}$ starts to rise). For $\sigma = 1.5$, the consumption smoothing motive is less important than for the baseline case $\sigma = 2.5$, so the volatility of consumption is higher.

<FIGURE 2>

The US Economy

We now consider the optimal R&D policy in the US. Obviously, as shown in section 3, the long run optimal subsidy rate $(s_A^{opt})^* = 0.815$ (for $\theta = 0.5$) exceeds the current one, $s_A = 0.066$, dramatically. When we calculate from Proposition 1 the steady state values of the adjusted knowledge level and adjusted per capita capital level, respectively, for $s_A = 0.066$ and compare the results with those when assuming $s_A = 0.815$, we find that the current knowledge stock and capital stock are merely at 5.4 percent and 6.3 percent of the long run optimal levels, respectively (when assuming that the US is currently in steady state). That is, in the US, $\tilde{A}^*/(\tilde{A}^{opt})^* = 0.054$ and $\tilde{k}^*/(\tilde{k}^{opt})^* = 0.063$.

Starting from these initial conditions, a policy reform which implements the dynamically optimal R&D subsidy is characterized as follows, shown in Fig. 3 (solid lines). The R&D subsidy rate, $s_A^{opt} = 1 - \tau^{opt}$, should initially jump upwards significantly (to about 83.6 percent) and then slightly decrease over time to $(s_A^{opt})^*$ (panel (a)). Again, the change in s_A^{opt} over time is small, despite the fact that we start far away from the long run optimum. Also the fraction of labor devoted to R&D (initially at about 4.2 percent under current R&D subsidization in the US), l^A, should jump upwards dramatically and then decrease considerably over time towards 18.2 percent (panel (b)). Thus, in steady state, the US should devote about four times as much labor to R&D than at present. The importance of knowledge accumulation for the growth process in the model is so high, that the initial savings rate should be slightly negative in the beginning under the optimal resource allocation (panel (d)). Overall,

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adjusted per capita consumption ($\tilde{c}$) increases significantly during the transition to the optimal steady state level $(\tilde{c}^{opt})^*$ (panel (c)). Notably, there is a long transition towards the new steady state with a half life of more than 200 years.

With respect to the welfare gain when the dynamically optimal policy rather than the constrained steady state optimal R&D subsidy is implemented, the insights of the previous subsection are confirmed also for the US. Despite the dramatic R&D underinvestment in the US, which is suggested by the model, the welfare difference is equivalent to a permanent change in per capita consumption ($LOSS$) of 2.5 percent. This is much higher than in the previous thought experiments (Tab. 1). The higher figure results from starting much farther away from the optimal steady state. Again, the time paths for per capita consumption and the savings rate are strikingly similar in the first-best and the constrained-optimum case. The fraction of R&D labor under the first-best policy should be somewhat higher than in the constrained optimum, especially in the beginning, but the difference in the two paths is rather small.

<FIGURE 3>

Distortionary Taxation

So far we have assumed that the subsidies on R&D and capital costs are financed by lump sum taxes. We now consider the case where they are financed by linear taxes on bond yields and labor income. The tax rates are denoted by t_r and t_w, respectively. In this case, the intertemporal budget constraint of the representative household modifies from (4) to

$$\dot{a} = [(1 - t_r)r - n]\, a + (1 - t_w)w - c. \tag{37}$$

Moreover, the no-arbitrage condition on the capital market changes to

$$\frac{\dot{P}^A}{P^A} + \frac{\pi}{P^A} = (1 - \mathsf{t}_r)r. \tag{38}$$

That is, the after-tax interest rate (when bond yields are taxed) must equal the sum of capital gains and the dividend-price ratio. We keep the balanced budget assumption. Thus, the tax revenue per capita, $\mathsf{t}_r rk + \mathsf{t}_w w$, equals government expenses per capita, $s_K(r+\delta)k + s_A wl^A$, in each period. This means that the four tax/subsidy instruments cannot be set independently from each other. For instance, given investment subsidies s_K, s_A and bond yield tax t_r, the labor income tax rate t_w is set such that the government budget is balanced. It is easy to see that (38) is consistent with the intertemporal budget constraint of the household when bond yields are taxed. The after-tax income from asset holding of a household is $(1-\mathsf{t}_r)rk + \dot{P}^A A/N + \pi A/N$. Using $k = a - P^A A/N$, according to (13), this equals $(1 - \mathsf{t}_r)ra - \left[(1 - \mathsf{t}_r)r - \frac{\dot{P}^A}{P^A} - \frac{\pi}{P^A}\right] AP^A/N = (1 - \mathsf{t}_r)ra$, according to (38).

Analogously to the previous analysis, we employ notation $p^A = P^A/N$, $l^A = L^A/N$ and $\tilde{z} = z/N^{\frac{1-\theta}{1-\phi}}$ for $z \in \{k, A, w, a, c\}$. As a consequence of distortionary taxation, Proposition 1 modifies to

Proposition 3. (Dynamic system for market equilibrium with distortionary taxes)

(i) Under factor income taxation, the evolution of $\tilde{A}$, $\tilde{k}$, l^A, r, p^A, $\tilde{c}$, t_r, $\tilde{a}$, $\tilde{w}$ is

governed by (15), (17), (19), (20),

$$\frac{\dot{p}^A}{p^A} = (1 - \mathsf{t}_r)r - n - \frac{(\kappa - 1)(\alpha/\kappa)^{\frac{1}{1-\alpha}}(1 - l^A)}{[(1 - s_K)(r + \delta)]^{\frac{\alpha}{1-\alpha}}p^A}, \tag{39}$$

$$\frac{\dot{\tilde{c}}}{\tilde{c}} = \frac{(1 - \mathsf{t}_r)r - \rho}{\sigma} - g, \tag{40}$$

$$\mathsf{t}_r r\tilde{k} + \mathsf{t}_w \tilde{w} = s_K(r + \delta)\tilde{k} + s_A \tilde{w} l^A, \tag{41}$$

$$\tilde{a} = \tilde{k} + p^A \tilde{A}, \tag{42}$$

$$\tilde{w} = (1 - \alpha)\tilde{A}^{1-\alpha} \left(\frac{\tilde{k}}{1 - l^A}\right)^\alpha. \tag{43}$$

(ii) In the long run, there exists a unique balanced growth equilibrium, where k, y, c and A grow at rate $g = \frac{(1-\theta)n}{1-\phi}$ and

$$r = \frac{\sigma g + \rho}{1 - \mathsf{t}_r} \equiv r^{**}, \tag{44}$$

$$l^A = \frac{1}{\frac{\tau(1/\alpha - 1)(\sigma g + \rho - n)}{(1 - 1/\kappa)g} + 1} \equiv (l^A)^{**}\left[= (l^A)^*\right], \tag{45}$$

$$\tilde{A} = \left(\frac{\nu\left((l^A)^{**}\right)^{1-\theta}}{g}\right)^{\frac{1}{1-\phi}} \equiv \tilde{A}^{**}\left[= \tilde{A}^*\right], \tag{46}$$

$$p^A = \frac{(\kappa - 1)(\alpha/\kappa)^{\frac{1}{1-\alpha}}(1 - (l^A)^{**})}{[(r^{**} + \delta)(1 - s_K)]^{\frac{\alpha}{1-\alpha}}(\sigma g + \rho - n)} \equiv (p^A)^{**}, \tag{47}$$

$$\tilde{k} = \tilde{A}^*(1 - (l^A)^{**})\left(\frac{\alpha}{\kappa(1 - s_K)(r^{**} + \delta)}\right)^{\frac{1}{1-\alpha}} \equiv \tilde{k}^{**}, \tag{48}$$

$$\tilde{c} = (\tilde{k}^{**})^\alpha(\tilde{A}^*)^{1-\alpha}(1 - (l^A)^{**})^{1-\alpha} - (\delta + n + g)\tilde{k}^{**} \equiv \tilde{c}^{**}, \tag{49}$$

$$\tilde{a} = \tilde{k}^{**} + (p^A)^{**}\tilde{A}^{**} \equiv \tilde{a}^{**}, \tag{50}$$

$$\tilde{w} = (1 - \alpha)\left(\tilde{A}^{**}\right)^{1-\alpha}\left(\frac{\tilde{k}^{**}}{1 - (l^A)^{**}}\right)^\alpha \equiv \tilde{w}^{**}, \tag{51}$$

$$\mathsf{t}_r r^{**}\tilde{k}^{**} + \mathsf{t}_w \tilde{w}^{**} = s_K(r^{**} + \delta)\tilde{k}^{**} + s_A \tilde{w}^{**}(l^A)^{**}. \tag{52}$$

The savings and investment rate is given by

$$\mathfrak{s} = \frac{\alpha(n + g + \delta)}{\kappa(1 - s_K)\left(\frac{\sigma g + \rho}{1 - \mathfrak{t}_r} + \delta\right)} \equiv \mathfrak{s}^{**}. \tag{53}$$

Proof. See Appendix. ■

In addition to the government budget constraint, which can be written as in (41), the capital income tax rate appears only in the differential equations which are associated with the intertemporal household budget constraint (37), i.e. the modified Keynes-Ramsey rule (40), and (39) which is associated with financial market clearing condition (38). The modified steady state values (44) and (47) follow directly from the dynamic system in Proposition 3. The other expressions are analogous to the case of lump sum taxation in Proposition 1. The labor income tax rate affects the equilibrium only through the government budget constraint, i.e. is non-distortionary.[22]

Equ. (50), (51) and (52) must obviously hold in steady state, according to (42), (43) and (41), respectively. (42) comes from the capital market clearing condition (13), $Na = K + P^A A$. Equ. (43) simply says that the wage rate, w, equals the marginal productivity of labor in the manufacturing sector; we can use (28) and $l^Y = 1 - l^A$ from (14) to derive the latter. A higher long run equilibrium capital tax rate $\mathfrak{t}_r$, reduces the steady state savings and investment rate, $\mathfrak{s}^{**}$. This result simply reflects the standard insight from in infinite-horizon models that capital income taxation depresses capital accumulation. Consequently, as we show in the next Proposition, the optimal subsidy to capital costs of firms should increase in $\mathfrak{t}_r$ (for a given interest rate).

Proposition 4. (Social optimum) *Under factor income taxation, the first-best*

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[22]The result would change if we allowed for human capital accumulation. See Grossmann et al. (2010).

optimal evolution of the economy can be supported by setting the capital cost subsidy to

$$s_K = 1 - \frac{(1 - \mathsf{t}_r)r + \delta}{\kappa(r + \delta)} \equiv s_K^{opt} \tag{54}$$

together with R&D subsidy $s_A^{opt} = 1 - \tau^{opt}$, where τ^{opt} evolves according to (33). Terminal conditions (optimal long run subsidy rates) are given by $(s_A^{opt})^{**} := 1 - (\tau^{opt})^*$ (see (34)) and

$$s_K = 1 - \frac{\sigma g + \rho + \delta}{\kappa \left(\frac{\sigma g + \rho}{1 - \mathsf{t}_r} + \delta\right)} \equiv (s_K^{opt})^{**}. \tag{55}$$

Proof. See Appendix. ∎

According to (54), the optimal subsidy to capital costs is generally time-variant under factor income taxation, in contrast to the case of lump-sum taxation. As the distortion of capital accumulation is fully internalized by the optimal capital investment subsidy and the steady state allocation of labor is not affected by factor income taxes, according to Proposition 3 and 4, optimal steady state levels of $\tilde{k}^{**}$, $\tilde{w}^{**}$ and $(l^A)^*$ do not depend on factor income tax rates. Substituting r^{**} from (44) and $s_K = (s_K^{opt})^{**}$ from (54) into (52) yields

$$\mathsf{t}_w = \left(1 - \frac{1}{\kappa}\right)(\sigma g + \rho + \delta)\frac{\tilde{k}^{**}}{\tilde{w}^{**}} + (s_A^{opt})^{**}(l^A)^{**} \equiv (\mathsf{t}_w^{opt})^{**}. \tag{56}$$

Thus, in steady state, the labor income tax rate which balances the government budget, $(\mathsf{t}_w^{opt})^{**}$, is independent of the tax rate on bond yields, t_r.

Similarly to the case of lump sum taxation, the loss in intertemporal welfare ($LOSS$) under factor income taxation when implementing the optimal long run subsidy rates, now for R&D *and* capital costs, rather than the dynamically policy program is small. For the baseline parameter set, $LOSS$ is 0.56 *per mill* when starting at a knowledge stock of 50 percent of the optimal steady state but 100 percent of the optimal cap-

ital stock (*Scenario (ii)* above), 0.09 *per mill* when initial conditions are vice versa (*Scenario (ii)*), 0.06 *per mill* when both state variables are 50 percent of the optimal steady state (*Scenario (iii)*) and 3 per cent when the knowledge stock and capital stock are merely at 5.4 percent and 6.3 percent of the long run optimal levels, respectively, which is the US case (assuming that the US is currently in steady state). These figures are only slightly higher than under lump sum taxation. Moreover, like in the case of lump sum taxation, $LOSS$ does not vary substantially under alternative parameter sets (sensitivity analysis available upon request).

Of course, in an alternative framework where all taxes are distortionary (e.g. with endogenous human capital accumulation the labor income tax would be distortionary as well), the first best allocation could not be implemented. In this case, one would have to analyze a "Ramsey-type" second-best policy problem, which, however, is beyond the scope of the present paper. This important issue in the context of dynamically optimal growth policy is left for future research.

Conclusion

We characterized the time path of first-best (i.e. time-varying) R&D subsidization in a semi-endogenous growth model and compared the allocative impact to the one arising from implementing the optimal steady state (i.e. time-invariant) policy from the start. We find that the differences in the time paths of per capita consumption and the allocation variables which result from the comparison between the first-best and the constrained optimal steady state policy are rather small. Our results suggest that the optimal R&D subsidization should change little over time, even when the economy starts far away from its socially optimal steady state. As a result, the welfare loss from a possible political constraint to use time-invariant policies is generally small. This insight is striking, and at a first glance surprising, given the slow speed of convergence

to the steady state in R&D-based endogenous growth models and given the large R&D underinvestment gap suggested by our stylized model. The economic reasoning for this observation lies in the standard consumption-smoothing motive which prevents the social planner to sacrifice a high level of consumption in the early transition phase.

The results regarding R&D subsidization do not hinge on whether investment subsidies are financed by lump sum taxation or factor income taxation. By contrast, the optimal subsidy on capital costs, which corrects inefficiency of capital accumulation from goods market power, is time-varying under taxation of bond yields, but not when subsidies are financed by lump sum taxes.

Future research should investigate optimal growth policy in alternative models to obtain a robust picture. Although our paper suggests that policy makers may focus on policy rates which are optimal in the long run, it is necessary to substantiate also this conclusion in alternative frameworks.

Acknowledgements

Appendix

Proof of Proposition 1: The current-value Hamiltonian which corresponds to the household optimization problem (12) is given by

$$\mathcal{H} = \frac{c^{1-\sigma} - 1}{1 - \sigma} + \lambda \left((r - n)a + w - c - T \right), \tag{57}$$

where λ is the co-state-variable associated with constraint (4). Necessary optimality conditions are $\partial \mathcal{H}/\partial c = 0$, $\dot{\lambda} = (\rho - n)\lambda - \partial \mathcal{H}/\partial a$, and the corresponding transversality condition. Thus,

$$\lambda = c^{-\sigma}, \text{ i.e. } \frac{\dot{\lambda}}{\lambda} = -\sigma \frac{\dot{c}}{c}, \tag{58}$$

$$\frac{\dot{\lambda}}{\lambda} = \rho - r, \tag{59}$$

$$\lim_{t \to \infty} \lambda_t e^{-(\rho-n)t} a_t = 0. \tag{60}$$

Combining (58) with (59), we obtain the standard Keynes-Ramsey rule

$$\frac{\dot{c}}{c} = \frac{r - \rho}{\sigma}. \tag{61}$$

Using $\dot{N}/N = n$ together with definitions $\tilde{c} = c/N^{\frac{1-\theta}{1-\phi}}$ and $g = \frac{(1-\theta)n}{1-\phi}$ confirms (18). In a similar fashion, (15) can be derived from (2).

According to Walras' law, the goods market is in equilibrium, i.e. $\dot{K} = Y - Nc - \delta K$. Thus, the capital stock per capita ($k = K/N$) evolves according to

$$\dot{k} = y - (\delta + n)k - c. \tag{62}$$

Using $x_i = K/A$ for all i in production function (1) gives us for per capita output ($y = Y/N$) the expression

$$y = k^\alpha (Al^Y)^{1-\alpha}, \tag{63}$$

where $l^Y = L^Y/N$ has been used. Combining (62) and (63) as well as using $l^Y = 1 - l^A$ from (14), $\dot{N}/N = n$, $\tilde{k} = k/N^{\frac{1-\theta}{1-\phi}}$ and $g = \frac{(1-\theta)n}{1-\phi}$ confirms (17). From (9), we find in addition that

$$r = \frac{\alpha}{\kappa(1 - s_K)} \frac{y}{k} - \delta. \tag{64}$$

Substituting (63) into (64) and using that $A/k = \tilde{A}/\tilde{k}$ then confirms (19).

Next, substitute (6) and (7) into (5) to obtain the following expression for the profit of each intermediate goods producer i:

$$\pi_i = \pi = (\kappa - 1)\left(\frac{\alpha}{\kappa}\right)^{\frac{1}{1-\alpha}}\left[(1-s_K)(r+\delta)\right]^{-\frac{\alpha}{1-\alpha}} L^Y. \tag{65}$$

Now recall definition $p^A = P^A/N$ as well as $L^Y/N = l^Y = 1 - l^A$ from (14) to rewrite (10) such as to confirm (16).

Since final goods producers take the wage rate as given, we have $w = (1-\alpha)Y/L^Y$. Thus,

$$w = (1-\alpha)A^{1-\alpha}\left(\frac{k}{l^Y}\right)^\alpha, \tag{66}$$

according to (63) and the fact that $Y/L^Y = y/l^Y$. Moreover, due to free entry in the R&D sector, in equilibrium, $\Pi = 0$ holds, i.e. $p^A \nu A^\phi N^{1-\theta}(l^A)^{-\theta} = \tau w$, according to (2) and (11). Inserting (66) implies

$$(1-\alpha)A^{1-\alpha}\left(\frac{k}{l^Y}\right)^\alpha = \frac{p^A \nu A^\phi N^{1-\theta}(l^A)^{-\theta}}{\tau}. \tag{67}$$

Using the definitions of $\tilde{k}$ and $\tilde{A}$ (thus, $A^{\phi-1}N^{1-\theta} = \tilde{A}^{\phi-1}$), we then obtain

$$(1-\alpha)\left(\frac{\tilde{k}}{\tilde{A}l^Y}\right)^\alpha = \frac{p^A \nu \tilde{A}^{\phi-1}(l^A)^{-\theta}}{\tau}. \tag{68}$$

Substituting $l^Y = 1 - l^A$ into (68) confirms (20). This concludes the proof of part (i).

To prove part (ii), we suspect (and later confirm) that in steady state, c grows with the same rate (g) as A, i.e. $\dot{\tilde{c}} = 0$. Using this in (18) confirms (21). Next, set $\dot{p}^A = 0$ in (16) to find

$$p^A = \frac{(\kappa - 1)(\alpha/\kappa)^{\frac{1}{1-\alpha}}(1-l^A)}{(r-n)[(1-s_K)(r+\delta)]^{\frac{\alpha}{1-\alpha}}}. \tag{69}$$

Moreover, substituting (8) into $w = (1-\alpha)Y/L^Y$ implies

$$\frac{w}{A} = (1-\alpha)\left(\frac{\alpha}{\kappa(1-s_K)(r+\delta)}\right)^{\frac{\alpha}{1-\alpha}}. \tag{70}$$

We know that with a time-invariant labor allocation in steady state knowledge stock A grows with rate g. From condition $\Pi = 0$, i.e. $p^A \dot{A} = \tau w l^A$, we find that $\dot{A}/A = g$ implies

$$p^A g = l^A \tau w / A. \tag{71}$$

Substituting both (69) and (70) into (71) and using (21) leads to (22). According to (15), $\dot{\tilde{A}} = 0$ implies (23). (24) immediately follows from (69). (25) follows from rearranging (19). To confirm (26), set $\dot{\tilde{k}} = 0$ in (17).

The savings rate (and investment share) is given by

$$\mathfrak{s} = (\dot{K} + \delta K)/Y = (\dot{K}/K + \delta)K/Y. \tag{72}$$

As $\dot{\tilde{k}} = 0$, we have $\dot{K}/K = n + g$ in steady state. Substituting this and (9) into (72) and using (21) confirms (27).

Finally, using $\dot{\lambda}/\lambda = -\sigma g$ from (58) and $\dot{c}/c = g$, we find that if a grows with rate g in the long run, the transversality condition (60) holds under assumption (A1). To see this, use from (13) that $a = k + p^A A$. Thus, in steady state, a grows at the same rate g as k and A.[23] This concludes the proof. ∎

Proof of Proposition 2: The current-value Hamiltonian which corresponds to

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[23] Also per capita lump sum tax T grows at rate g in the long run. To see this, consider the government budget constraint $T = s_K(r+\delta)k + s_A w l^A$. Since w grows at the same rate as A, according to (70), and r is time-invariant, we see that $\dot{T}/T = \dot{w}/w = \dot{k}/k = g$ in steady state.

the social planning problem (31) is given by

$$\mathcal{H} = \frac{c^{1-\sigma}-1}{1-\sigma} + \lambda_k \overbrace{(k^\alpha(Al^Y)^{1-\alpha}}^{\dot{k}} - (\delta+n)k - c) + \lambda_A \overbrace{\nu A^\phi N^{1-\theta} \underbrace{(1-l^Y)^{1-\theta}}_{=l^A}}^{=\dot{A}}, \quad (73)$$

where λ_k and λ_A are co-state variables associated with constraints (29) and (30), respectively. Necessary optimality conditions are $\partial\mathcal{H}/\partial c = \partial\mathcal{H}/\partial l^Y = 0$ (control variables), $\dot{\lambda}_z = (\rho-n)\lambda_z - \partial\mathcal{H}/\partial z$ for $z \in \{k,A\}$ (state variables), and the corresponding transversality conditions. Thus,

$$\lambda_k = c^{-\sigma}, \text{ i.e. } \frac{\dot{\lambda}_k}{\lambda_k} = -\sigma\frac{\dot{c}}{c}, \quad (74)$$

$$(1-\alpha)A^{1-\alpha}\left(\frac{k}{l^Y}\right)^\alpha = \frac{\lambda_A}{\lambda_k}(1-\theta)\underbrace{\nu A^\phi N^{1-\theta}(l^A)^{-\theta}}_{=\dot{A}/l^A}, \quad (75)$$

$$\frac{\dot{\lambda}_k}{\lambda_k} = \rho - \alpha\left(\frac{Al^Y}{k}\right)^{1-\alpha} + \delta, \quad (76)$$

$$\frac{\dot{\lambda}_A}{\lambda_A} = \rho - n - \frac{\lambda_k}{\lambda_A}(1-\alpha)\underbrace{\left(\frac{k}{A}\right)^\alpha (l^Y)^{1-\alpha}}_{=y/A} - \phi\frac{\dot{A}}{A} \quad (77)$$

$$\lim_{t\to\infty} \lambda_{z,t}e^{-(\rho-n)t}z_t = 0, \ z \in \{k,A\}. \quad (78)$$

($\lambda_{z,t}$ denotes the co-state variable associated with state variable z at time t.)

To find the optimal capital cost subsidy, first note that from (58) and (74) that we must have $\lambda = \lambda_k$ in social optimum; thus, according to (59) and (76),

$$r + \delta = \alpha\left(\frac{Al^Y}{k}\right)^{1-\alpha}. \quad (79)$$

Comparing (79) with (19), by using $l^Y = 1 - l^A$ and the definitions of $\tilde{c}, \tilde{k}, \tilde{A}$, we find that $\kappa(1-s_K) = 1$ must hold in social optimum at all times, which is equivalent to

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(32).

Next, note from (67) and (75) that a R&D subsidy which implements the social optimum must fulfill

$$p^A = (1-\theta)\tau\frac{\lambda_A}{\lambda_k}, \text{ i.e.} \tag{80}$$

$$\frac{\dot{\tau}}{\tau} = \frac{\dot{p}^A}{p^A} - \left(\frac{\dot{\lambda}_A}{\lambda_A} - \frac{\dot{\lambda}_k}{\lambda_k}\right). \tag{81}$$

Moreover, substituting optimality conditions $1 - s_K = 1/\kappa$ and (80) into (16), we find

$$\frac{\dot{p}^A}{p^A} = r - n - \frac{(1-1/\kappa)\alpha(1-l^A)}{(Al^Y/k)^\alpha\,\tau(1-\theta)}\frac{\lambda_k}{\lambda_A}. \tag{82}$$

Rewriting (75) to

$$\frac{\lambda_k}{\lambda_A} = \frac{(1-\theta)\left(Al^Y/k\right)^\alpha}{(1-\alpha)l^A}\frac{\dot{A}}{A} \tag{83}$$

and substituting into (82) leads to

$$\frac{\dot{p}^A}{p^A} = r - n - \frac{(1-1/\kappa)(1-l^A)}{\tau(1/\alpha-1)l^A}\frac{\dot{A}}{A}. \tag{84}$$

Moreover, combining (76) and (77) by subtracting both sides of the equations from each other and substituting (83), we have

$$\frac{\dot{\lambda}_A}{\lambda_A} - \frac{\dot{\lambda}_k}{\lambda_k} = \alpha\left(\frac{Al^Y}{k}\right)^{1-\alpha} - \delta - n - (1-\theta)\frac{l^Y}{l^A}\frac{\dot{A}}{A} - \phi\frac{\dot{A}}{A}. \tag{85}$$

Substituting (84) and (85) into (81) and making use of (79) and $l^Y = 1 - l^A$ then leads to

$$\frac{\dot{\tau}}{\tau} = \left[\left(1 - \theta - \frac{1}{\tau}\frac{1-1/\kappa}{1/\alpha-1}\right)\frac{1-l^A}{l^A} + \phi\right]\frac{\dot{A}}{A}. \tag{86}$$

From (30) and the definition of $\tilde{A}$ we find

$$\frac{\dot{A}}{A} = \nu \tilde{A}^{\phi-1} \left(l^A\right)^{1-\theta}. \tag{87}$$

Substituting (87) into (86) confirms (33).

It remains to confirm terminal condition (34), i.e. the optimal long run R&D policy. We seek for a steady state where A, k and c all grow at rate g and $\dot{l}^Y = \dot{l}^A = \dot{\tau} = 0$. Setting $\dot{\tau}^{opt} = 0$ in (33) implies that, in long run social optimum,

$$\left(1 - \theta - \frac{1}{\tau^{opt}} \frac{1 - 1/\kappa}{1/\alpha - 1}\right) \left(\frac{1}{l^A} - 1\right) + \phi = 0. \tag{88}$$

To infer (34) from (88) we need to find the steady state value for the fraction of R&D labor, l^A. From (74), $\dot{\lambda}_k/\lambda_k = -\sigma g$; combining with (76) implies

$$\alpha \left(\frac{A l^Y}{k}\right)^{1-\alpha} - \delta = \sigma g + \rho. \tag{89}$$

From (83) and $\dot{A}/A = g$, together with the property that $A l^Y / k$ are l^A are constant in the long run, we find $\dot{\lambda}_k/\lambda_k = \dot{\lambda}_A/\lambda_A$. Using $\dot{\lambda}_k/\lambda_k = \dot{\lambda}_A/\lambda_A$, $\dot{A}/A = g$, (89) and $l^Y = 1 - l^A$ in (85) we can solve for l^A. Doing so and using $(1 - \phi)g = (1 - \theta)n$ from the definition of g in (A1) implies

$$l^A = \frac{1}{\frac{(\sigma-1)g+\rho-\theta n}{(1-\theta)g} + 1}. \tag{90}$$

Substituting (90) into (88) and using $(1 - \phi)g = (1 - \theta)n$ confirms (34).

From (74) and $\dot{c}/c = g$, we have $\dot{\lambda}_k/\lambda_k = -\sigma g$. Using that $\dot{\lambda}_k/\lambda_k = \dot{\lambda}_A/\lambda_A = -\sigma g$ for $t \to \infty$ and $\dot{k}/k = \dot{A}/A = g$, transversality condition (78) is fulfilled under assumption (A1) for both state variables, k and A.

So far we have shown that the policy mix in Proposition 2 is necessary for a first-

best optimum. To show that it is also sufficient, we need to prove that under this policy mix, the market equilibrium is the same as the social planning optimum.

From (29) we obtain

$$\frac{\dot{k}}{k} = \left(\frac{Al^Y}{k}\right)^{1-\alpha} - \delta - n - \frac{\tilde{c}}{\tilde{k}}, \tag{91}$$

which coincides with (17) by using $l^Y = 1 - l^A$ and the definitions of $\tilde{c}$, $\tilde{k}$, $\tilde{A}$. Similarly, (87) coincides with (15) and combining (74) with (76) leads to (18) when using (79). Finally, combining (80) with (83) and using (87) leads to (20). This concludes the proof. ∎

Proof of Proposition 3: First, note that we can use analogous reasoning as in the proof of Proposition 1 to see that (15), (17), (19), (20) still hold. Moreover, (41), (42) and (43) were confirmed in the text.

The current-value Hamiltonian which corresponds to the household optimization problem now reads

$$\mathcal{H} = \frac{c^{1-\sigma} - 1}{1 - \sigma} + \lambda \left([(1 - \mathsf{t}_r)r - n]\, a + (1 - \mathsf{t}_w)w - c \right), \tag{92}$$

Thus, we can replace (59) by

$$\frac{\dot{\lambda}}{\lambda} = \rho - (1 - \mathsf{t}_r)r, \tag{93}$$

which leads to the modified Keynes-Ramsey rule

$$\frac{\dot{c}}{c} = \frac{(1 - \mathsf{t}_r)r - \rho}{\sigma}. \tag{94}$$

Using the definition of $\tilde{c}$ confirms (40). Moreover, substituting (65) into (38) and using $p^A = P^A/N$ as well as $L^Y/N = l^Y = 1 - l^A$ confirms (39). This concludes the proof of part (i).

Also the derivation of the steady state (part (ii) of Proposition 3) is analogous to the proof of Proposition 1. Using $\dot{\tilde{c}} = 0$ in (40) confirms (44) and using $\dot{p}^A = 0$ in (39) together with (44) confirms (47). Substituting both (47) and (70) into (71) and using (44) confirms that $l^A = (l^A)^*$ in steady state. Moreover, combining (15) and $\dot{\tilde{A}} = 0$ implies $A = A^*$. Substituting $\dot{K}/K = n + g$ and (9) into (72) and using (44) confirms (53). The reminder of the proof is obvious. ∎

Proof of Proposition 4: The result is proven analogously to Proposition 2. First, to find the optimal capital cost subsidy, note that using $\lambda = \lambda_k$ and combining (76) with (93) implies

$$(1 - \mathsf{t}_r)r + \delta = \alpha \left(\frac{Al^Y}{k} \right)^{1-\alpha}. \tag{95}$$

Moreover, (19) and $l^Y = 1 - l^A$ imply

$$(r + \delta)\kappa(1 - s_K) = \alpha \left(\frac{Al^Y}{k} \right)^{1-\alpha}. \tag{96}$$

Equating the left-hand sides of (95) and (96) confirms (54). Using $r = r^{**}$ as given by (44) also confirms (55).

Next, substitute $(r + \delta)(1 - s_K) = (\alpha/\kappa)\left(Al^Y/k\right)^{1-\alpha}$ from (96) into (39) and use both (80) and (83) to find

$$\frac{\dot{p}^A}{p^A} = (1 - \mathsf{t}_r)r - n - \frac{(1 - 1/\kappa)(1 - l^A)}{\tau(1/\alpha - 1)l^A} \frac{\dot{A}}{A}. \tag{97}$$

Substituting (85) and (97) into (81) and using (95) confirms that (86) still holds. As also (87) still applies, we also confirm (33) for the long run. ∎

Measuring Welfare Differences: We first quantify the welfare difference, denoted by ΔU, between implementing the dynamically optimal policy mix and the optimal steady state policy mix, taking into account the whole transition path to calculate

the respective welfare levels. We then use ΔU to calculate the hypothetical permanent percentage loss of the optimal steady state per capita consumption of not implementing the dynamically first best policy. Formally, recall that in the long run adjusted per capita consumption, $\tilde{c} = c/N^{\frac{1-\theta}{1-\phi}}$, is stationary. Thus, per capita consumption c grows with rate $g = \frac{(1-\theta)n}{1-\phi}$. Thus, in the long run under the optimal policy, $c = e^{gt}(\tilde{c}^{opt})^*$. Denote the hypothetical loss in adjusted steady state per capita consumption from not implementing the dynamically optimal R&D subsidy by $\Delta\tilde{c}$. Then we have:

$$\Delta U = \int_0^\infty \frac{((\tilde{c}^{opt})^* e^{gt})^{1-\sigma} - 1}{1-\sigma} e^{-(\rho-n)t} dt - \int_0^\infty \frac{(((\tilde{c}^{opt})^* - \Delta\tilde{c})e^{gt})^{1-\sigma} - 1}{1-\sigma} e^{-(\rho-n)t} dt \tag{98}$$

which we can rewrite to find, as reported in the text and in Tab. 1 and 2, the percentage loss,

$$LOSS := \frac{\Delta\tilde{c}}{(\tilde{c}^{opt})^*} = 1 - \frac{[((\tilde{c}^{opt})^*)^{1-\sigma} + \Delta U(\sigma-1)(\rho-n+(\sigma-1)g)]^{\frac{1}{1-\sigma}}}{(\tilde{c}^{opt})^*}. \tag{99}$$

Numerical evaluation - the relaxation algorithm: Trimborn, Koch and Steger (2008) have developed a powerful numerical evaluation method in order to simulate highly-dimensional, non-linear systems of differential equations even when starting far away from the steady state. The principle of the method is to construct a large set of non-linear algebraic equations. The root of these equation represents the solution trajectory. The set of equations is obtained by discretizaton of the differential equations on a mesh of points in time augmented by conditions representing initial and final boundary conditions. Then, the whole set of equations is solved simultaneously. The advantage of this procedure is that it calculates the solution taking non-linearities of the model into account. Therefore, it allows to obtain trajectories representing transitional dynamics up to an arbitrarily small error. This property is especially useful for calculating utility, for which a high degree of accuracy is needed to receive a

precise ranking of policy scenarios.

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Figure Legends

Figure 1: Transitional dynamics (solid lines: first-best solution, dashed lines: constrained optimal policy). Scenario (i): $\tilde{A}_0 = 0.5\tilde{A}^*$ and $\tilde{k}_0 = \tilde{k}^*$; Scenario (ii): $\tilde{A}_0 = \tilde{A}^*$ and $\tilde{k}_0 = 0.5\tilde{k}^*$; Scenario (iii): $\tilde{A}_0 = 0.5\tilde{A}^*$ and $\tilde{k}_0 = 0.5\tilde{k}^*$.

Figure 2: Time paths of $\tilde{c}$ for baseline set of parameters (i.e. $\sigma = 2.5$, $\rho = 0.02$) and $\sigma = 1.5$, $\rho = 0.04$ under the two policy schemes (solid lines: dynamically optimal policy; dashed line: constrained optimal policy under baseline; dotted line: constrained optimal policy under $\sigma = 1.5$, $\rho = 0.04$).

Figure 3: Transitional dynamics for the US economy (solid lines: first-best solution, dashed lines: constrained optimal policy). In panel (b) and (d) the circles indicate the initial steady state values under the status quo policies in the US.

Tables

Parameter set	LOSS in (i)	LOSS in (ii)	LOSS in (iii)
Baseline	0.37	0.03	0.05
$\sigma = 1.5$, $\rho = 0.04$	0.33	0.01	0.17
$\sigma = 3$, $\rho = 0.01$	0.42	0.05	0.02
$\sigma = 1.5 \Rightarrow r^* = 0.05$	0.34	0.02	0.16
$\rho = 0.01 \Rightarrow r^* = 0.06$	0.31	0.04	0.02
$\theta = 0.7$, $\phi = 0.85$	0.18	0.02	0.01
$\theta = 0.3$, $\phi = 0.4$	0.59	0.04	0.01
$\kappa = 1.1$	0.37	0.03	0.05

Table 1. The welfare loss from implementing the long run rather than the dynamically optimal R&D policy under lump sum taxation, in per mill.

Notes. We assume $s_K = (s_K^{opt})^* = 1 - 1/\kappa$ throughout. *Scenario (i):* $\tilde{A}_0 = 0.5(\tilde{A}^{opt})^*$, $\tilde{k}_0 = (\tilde{k}^{opt})^*$, *Scenario (ii):* $\tilde{A}_0 = (\tilde{A}^{opt})^*$, $\tilde{k}_0 = 0.5(\tilde{k}^{opt})^*$, *Scenario (iii):* $\tilde{A}_0 = 0.5(\tilde{A}^{opt})^*$, $\tilde{k}_0 = 0.5(\tilde{k}^{opt})^*$. The baseline parameter set is $g = 0.02$, $n = 0.01$, $\delta = 0.04$, $\alpha = 0.33$, $\kappa = 4/3$, $\sigma = 2.5$, $\rho = 0.02$, $\theta = 0.5$, $\phi = 0.75$. The indicated parameter(s) display the respective deviations from the baseline set of parameters.